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Confinement in 4D Yang-Mills theories from non-critical Type 0 string theory

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Abstract: We study five dimensional non critical type 0 string theory and its correspondence to non supersymmetric Yang Mills theory in four dimensions. We solve the equations of motion of the low energy effective action and identify a class of solutions that translates into a confining behavior in the IR region of the dual gauge theories. In particular we identify a setup which is dual to pure SU(N) Yang-Mills theory. Possible flows of the solutions to the UV region are discussed. The validity of the solutions and potential sub-leading string corrections are also discussed.

Keywords: D-branes, Brane Dynamics in Gauge Theories.

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1. Introduction

It is well known that the original motivation behind string theory was the search for the theory of the strong interactions. In spite of the fact that string theory "drifted" to a very different domain, the original quest of a stringy description of Yang Mills theory is still an important challenge.

A step forward in this direction was made by Polyakov who argued [1, 2] that this string theory is associated with a Liouville theory with a curved fifth dimension. Recently, the seminal work of Maldacena [3] followed by [4] and [5] provoked a dramatic breakthrough in the interplay between string theory and supersymmetric conformal gauge theory in the large N limit. A natural question that has been raised following this development is whether similar insight from gravity and string theory can be also achieved about non-conformal and non-supersymmetric four dimensional gauge theories. In particular whether one can merge the ideas of Polyakov with those of the string/gauge duality.

Indeed this question attracted recently a lot of attention mainly along the direction of a critical type 0 string theory [6, 7, 8, 9, 10, 11, 12, 13], but also in a non-critical setting [14, 15]. A key point in the implementation of a consistent bosonic string theory is rendering the tachyon field into a "good tachyon", namely, shifting the value of m^2 to a positive one. Klebanov and Tseytlin [6] showed that in the background of D-branes, the coupling of the tachyon to the R-R flux can remove the tachyon instability. Next it was shown [7, 8] that the UV behavior of asymptotically free gauge theories can be extracted from the gravity description of the theories. The Infra-Red behavior of these theories was first considered in [8], where the IR fixed point of 4d Yang-Mills with 6 massless adjoint scalars was identified as the asymptotically $AdS_5 \times S^5$ metric. It was then argued [12] that the adjoint scalars would acquire mass through loop corrections and that the generic behavior, in the IR, is that of a pure Yang-Mills theory. Accordingly, a confining solution of the gravity equations of motion was found. Another interesting result was the identification of a large N non-supersymmetric CFT [10]. The theory is a $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge theory with 6 adjoint scalars and 4 bifundamental Weyl fermions in the $(\bar{N}, N) \oplus (N, \bar{N})$ representation.

In this paper we follow the direction paved by Polyakov and discuss non-critical type 0 string theory in five dimensions. The analysis of the non critical theory faces more problems than the critical one, however, it has the advantage that it does not include degrees of freedoms which are associated with the extra dimensions. Hence it is more closely related to the pure YM theory. Our main result is that there are solutions to the equations of motion, derived from the effective action, that correspond to confinement in the dual gauge theory. The notion of confinement here means that a Wilson loop deduced from a Nambu Goto action in the background of the metric solution admits an area law behavior. The solutions have non-zero measure in the space of solutions, though other solutions also exist. We also derive "flows" of the solutions that corresponds to the UV behavior of the gauge theory. We find two possible scenarios: (i) Solutions that are connected to an AdS_5 solution which corresponds to a UV fixed point. (ii) Solutions in which the effective gauge coupling is asymptotically free. Unfortunately, since the analysis is based on numerical integration of differential equations, we are not able to extract the β function from the UV behavior of the Wilson loop. In the absence of flat directions we believe the solutions are associated with gauge theories that do not include (adjoint) scalars and hence those gauge theories are in a confining phase. Specifically, 5d type 0 string theory in the background of N D3 "electric" branes is conjectured to be dual to pure 4d Yang-Mills theor [2]. We also find certain evidence for a non critical analog (with no scalars and one bifundamental fermion) of the $\mathrm{SU}(N) \times \mathrm{SU}(N)$ gauge theory of [10].

Note that the notion of "electric" and "magnetic" branes refers here to charges with respect to two field strength five forms. Obviously, unlike the critical case [10], in 5d they are not duals of each other. The coupling of the tachyon to the R-R forms, which in this case is symmetric under $T \to -T$, leads to a confining solution evolving to the UV with a vanishing tachyon.

Beyond the extraction of solutions admitting "gauge dynamics", the purpose of our work is to identify the conditions required to assure the reliability of the solutions. We discuss the tadpole cancelation, the shift of the tachyonic instability, the consistency of the non-critical string, higher order string perturbations, curvature corrections and the stability of the stack of N $D3$ branes.

The outline of the paper is as follows. We consider the degrees of freedom of the non critical type 0 theory in five dimensions and review its low energy effective action and the corresponding equations of motion in section 2. In section 3 we derive solutions to those equations. An exact AdS_5 solution, generic confining solutions and their flows to the UV region. In section 4 we discuss the conditions for the validity of the solutions. Section 5 is devoted to the field theory interpretation of the gravity solutions. This is addressed via the symmetries of the boundary theory, the gauge coupling, field content, Wilson loop, 't Hooft loop and Zig Zag invariance. The result of this work are summarized in section 6. In appendix A we discuss the asymptotic behavior of the solutions.

2. Non-critical Type 0 string theory

Let us start with the identification of the low energy degrees of freedom, namely, the massless (and tachyonic) states of the 5d type 0 string. The degrees of freedom in the bulk come from the close string sectors.

In the 5d non-critical string theory, the degrees of freedom are in the SO(3) representation. From the $(\mathrm{NS}-, \mathrm{NS}-)$ sector we get $0 \otimes 0 = 0$ which is the tachyon T. From the $(\mathrm{NS}+, \mathrm{NS}+)$ sector we get $1 \otimes 1 = 2 \oplus 1 \oplus 0$ which are the graviton G_{mn}, the anti-symmetric tensor B_{mn} and the dilaton Φ. The R sector vacuum is in the 1/2 representation of Spin(3) $\sim$ SU(2). From the (R, R) sector we get $\frac{1}{2} \otimes \frac{1}{2} = 1 + 0$, namely a 0 and a 1 forms. The 1- form of the (R, R) sector is associated with the 2-form field strength of the $D0$ brane. The 0-form associated with a 1-form field strength of the $D-1$ brane. By dualizing the $(p+2)$ field strength of the Dp brane we can get the $5-(p+2) = (q+2)$ field strength of the $Dq = D(1-p)$ brane. This gives us the $D1$ and the $D2$ branes. We cannot get the $D3$ branes directly from the (R, R) sector. Similar thing happens in 10d string theory where one cannot get the $D8$ branes from the (R, R) sector. We assume that the $D3$ branes exist because we can get them from $D2$ branes using T-duality.

The diagonal GSO projection in the critical string includes the states $(\mathrm{R}+, \mathrm{R}-) \oplus (\mathrm{R}-, \mathrm{R}+)$ in type 0A and $(\mathrm{R}+, \mathrm{R}+) \oplus (\mathrm{R}-, \mathrm{R}-)$ in type 0B. In the 5d non-critical string there is no chirality and hence one cannot assign $+,-$ to the Ramond ground state. The diagonal GSO projection, does not determine the multiplicity of the (R,R) sector. However, from the requirement of modular invariance of the torus partition function, we believe that the (R,R) sector has to be doubled. Assuming there is doubling of the (R,R) sector we would have two sets of D-branes, electric and magnetic.

The low energy type 0 effective action built from the $(\mathrm{NS},\mathrm{NS})$ and (R,R) sectors was derived in [6] for any dimension, both critical and non-critical. We briefly summarize its features using their notations. The supergravity action from the $(\mathrm{NS}+,\mathrm{NS}+)$ sector is the same as the type IIB action

$$S=-2\int d^Dx\sqrt{G}e^{-2\Phi}\left[c_0+R+4(\partial_n\Phi)^2-\frac{1}{12}H^2_{mnk}\right],$$

where $c_0=\frac{10-D}{\alpha'}$ is the central charge term that appears for non critical string theories. The action of the tachyon that comes from the $(\mathrm{NS}-,\mathrm{NS}-)$ sector is

$$S=\int d^Dx\sqrt{G}e^{-2\Phi}\left(\frac{1}{2}G^{mn}\partial_mT\partial_nT+\frac{1}{2}m^2T^2\right),$$

where $m^2=(2-D)/4\alpha'$ is the mass of the tachyon. The leading R-R terms in the action are

$$S=\int d^Dx\sqrt{G}h^{-1}_{(n+1)}(T)|F_{n+1}|^2,$$

where F_{n+1} is the field strength of the R-R field and $h_{(n+1)}(T)$ describes the coupling of the tachyon to the R-R field.

For the theory on the background of D-Branes we use the following ansatz for the metric and the R-R electric field

$$\begin{aligned}ds^2&=d\tau^2+e^{2\lambda(\tau)}dx^2_\mu+e^{2\nu(\tau)}d\Omega^2_k,\\ C_n&=A(\tau),\\ F_{n+1}&=A'(\tau),\end{aligned}$$

where x_μ are the coordinates of an n dimensional flat Minkowski space and Ω_k is a k dimensional sphere. The string theory is in $D=n+k+1$ dimensions. C_n is a n-form with all the indices in the Minkowski space, and F_{n+1} is a $(n+1)$-form where the extra index is in the τ direction. We also assume for the tachyon that $T=T(\tau)$.

Defining the function $\varphi\equiv 2\Phi-n\lambda-k\nu$ and plugging the metric into the string actions we obtain

$$\begin{aligned}S=-\int d\tau\Big(&e^{-\varphi}\left[c_0+k(k-1)e^{-2\nu}-n\lambda'^2-k\nu'^2+\varphi'^2-\frac{1}{4}T'^2-\frac{1}{4}m^2T^2\right]+\\ &+\frac{1}{4}e^{-n\lambda+k\nu}h^{-1}(T)A'^2\Big),\end{aligned}\tag{2.1}$$

Solving the equation of motion for $A(\tau)$ we obtain

$$A' = 2Qe^{n\lambda - k\nu}h(T)\,,$$

where Q is the charge of the R-R field.

In order to have a Toda-like mechanical system we define a new 'time' parameter ρ

$$d\rho = e^{\varphi}d\tau\,.$$

To get a four dimensional gauge theory with no scalars, we choose $n = 4$ and $k = 0$. Now the action is independent of ν. We also set $\alpha' = 1$. Instead of working with φ and λ, we work with the real dilaton Φ and with the function $\xi = \frac{1}{2}(\varphi + \lambda)$ which is the combination of φ and λ that diagonalize the action

$$\begin{aligned}
\varphi &= -\frac{2}{n-1}\Phi + \frac{2n}{n-1}\xi = -\frac{2}{3}\Phi + \frac{8}{3}\xi\,,\\
\lambda &= \frac{2}{n-1}\Phi - \frac{2}{n-1}\xi = \frac{2}{3}\Phi - \frac{2}{3}\xi\,.
\end{aligned}$$

The metric in the new variables is

$$ds^2 = e^{\frac{4}{3}\Phi - \frac{16}{3}\xi}d\rho^2 + e^{\frac{4}{3}\Phi - \frac{4}{3}\xi}dx_\mu^2\,, \tag{2.2}$$

and we get the action

$$\begin{aligned}
S &= \int d\rho\left[\frac{4}{3}\dot{\Phi}^2 - \frac{16}{3}\dot{\xi}^2 + \frac{1}{4}\dot{T}^2 - V(\Phi,\xi,T)\right],\\
V(\Phi,\xi,T) &= \left(5 + \frac{3}{16}T^2\right)e^{\frac{4}{3}\Phi - \frac{16}{3}\xi} - Q^2 h(T)e^{\frac{10}{3}\Phi - \frac{16}{3}\xi}\,.
\end{aligned}$$

The equations of motion from the action are

$$\ddot{\Phi} + \frac{1}{2}\left(5 + \frac{3}{16}T^2\right)e^{\frac{4}{3}\Phi - \frac{16}{3}\xi} - \frac{5}{4}Q^2 h(T)e^{\frac{10}{3}\Phi - \frac{16}{3}\xi} = 0\,, \tag{2.3}$$

$$\ddot{\xi} + \frac{1}{2}\left(5 + \frac{3}{16}T^2\right)e^{\frac{4}{3}\Phi - \frac{16}{3}\xi} - \frac{1}{2}Q^2 h(T)e^{\frac{10}{3}\Phi - \frac{16}{3}\xi} = 0\,, \tag{2.4}$$

$$\ddot{T} + \frac{3}{4}Te^{\frac{4}{3}\Phi - \frac{16}{3}\xi} - 2Q^2 h'(T)e^{\frac{10}{3}\Phi - \frac{16}{3}\xi} = 0\,. \tag{2.5}$$

To derive the full set of equations of motion of the string action one has to add the zero-energy, "Gauss-law" like, constraint

$$\frac{4}{3}\dot{\Phi}^2 - \frac{16}{3}\dot{\xi}^2 + \frac{1}{4}\dot{T}^2 + V(\Phi,\xi,T) = 0\,. \tag{2.6}$$

The function $h(T)$ that describes the coupling of the tachyon to the R-R field for N parallel D3 electric branes is [6]

$$\begin{aligned}
h(T) &= f^{-1}(T)\\
f(T) &= 1 + T + \frac{1}{2}T^2 + O(T^3) \approx e^T\,,
\end{aligned}$$

For the set of electric-magnetic D3 branes, when the electric and magnetic charges are the same we get [10]

$$h(T) = f(T) + f^{-1}(T)\,. \tag{2.7}$$

Note that the IR behavior of the dual gauge theory is dictated by the behavior of $\frac{4}{3}(\Phi - \xi) = 2\lambda$ which appears in the four dimensional space-time part of the metric (2.2). From the Wilson loop calculations we know that if λ has a unique minimum at a certain value of ρ then the theory will confine. Therefore it is useful to write the metric, the Hamiltonian and the equations of motions in terms of λ and Φ. The metric is

$$ds^2 = e^{-4\Phi+8\lambda} d\rho^2 + e^{2\lambda} dx_\mu^2\,. \tag{2.8}$$

The hamiltonian is

$$\begin{aligned} H = &-4\dot{\Phi}^2 + 16\dot{\Phi}\dot{\lambda} - 12\dot{\lambda}^2 + \frac{1}{4}\dot{T}^2 + \\ &+ \left(5 + \frac{3}{16}T^2\right) e^{-4\Phi+8\lambda} - Q^2 h(T) e^{-2\Phi+8\lambda} = 0\,, \end{aligned} \tag{2.9}$$

and the equations of motion are

$$\ddot{\Phi} + \frac{1}{2}\left(5 + \frac{3}{16}T^2\right) e^{-4\Phi+8\lambda} - \frac{5}{4}Q^2 h(T) e^{-2\Phi+8\lambda} = 0\,, \tag{2.10}$$

$$\ddot{\lambda} - \frac{1}{2}Q^2 h(T) e^{-2\Phi+8\lambda} = 0\,, \tag{2.11}$$

$$\ddot{T} + \frac{3}{4}T e^{-4\Phi+8\lambda} - 2Q^2 h'(T) e^{-2\Phi+8\lambda} = 0\,. \tag{2.12}$$

The charge of the R-R field is proportional to the number of branes $Q \sim N$. It is convenient to absorb this factor by the redefinition $\Phi = \tilde{\Phi} - \ln N$, $\lambda = \tilde{\lambda} - \frac{1}{2}\ln N$. This redefinition fixes the N dependence in the metric (2.8). The 4d space-time part is multiplied by $1/N$. Also, the curvature in the string frame does not depend on N. The Einstein frame in 5d in defined by $ds^2 = e^{\frac{4}{3}\Phi} ds_E^2$, therefore the curvature in the Einstein frame behaves as $1/N^{4/3}$.

3. Solutions of the equations of motion

In this section we derive configurations of the metric, dilaton and tachyon that solve the equations of motion, and as will be discussed in section 5 incorporate interesting gauge theory interpretation. We find an *exact* AdS_5 solution. In addition we write down solutions which later will be argued to describe the IR regime of the corresponding gauge theories and we use numerical analysis to flow to the UV regime.

We discuss both the electric branes system as well as the electric-magnetic system [6, 10]. The later is somewhat simpler since in this case $T = 0$ is a solution of the tachyon equation of motion (2.12). This is a consequence of the symmetry $T \to -T$ in $h(T)$ which results from the electric-magnetic symmetry of the problem.

3.1 The AdS_5 solution

An *exact* solution to (2.3), (2.4) and (2.5) can be found for a general function $h(T)$ by taking Φ and T to be constants, and solving the algebraic equations

$$\frac{1}{2}\left(5+\frac{3}{16}T^2\right)-\frac{5}{4}Q^2h(T)e^{2\Phi}=0\,,$$
$$\frac{3}{4}T-2Q^2h'(T)e^{2\Phi}=0\,.$$

Then the differential equation for ξ is solved exactly by $\xi\sim\frac{3}{8}\ln\rho$. Specifically, for the electric magnetic case (2.7), the solution is

$$\begin{aligned}T&=0\,,\\ \Phi&=\frac{1}{2}\ln\frac{1}{Q^2}\,,\\ \xi&=\frac{3}{8}\ln\rho+\frac{1}{8}\ln\frac{8}{Q^2}\,,\end{aligned}\tag{3.1}$$

and the metric corresponding to this solution is

$$ds^2=\frac{1}{4\rho^2}d\rho^2+\frac{1}{2^{1/2}Q}\frac{1}{\rho^{1/2}}dx_\mu^2\,,$$

which, upon the replacement $\rho=1/u^4$ can be brought to the familiar AdS_5 form

$$ds^2=4\frac{du^2}{u^2}+\frac{1}{2^{1/2}Q}u^2dx_\mu^2\,,\tag{3.2}$$

The corresponding curvature is

$$\alpha'\mathcal{R}=-5\,.$$

This solution, which has already appeared in [2], may seem to have a very different curvature than that of the $AdS_5\times S^5$ [3] $\alpha'\mathcal{R}\sim 1/\sqrt{g_sN}$. In fact it is of the same nature since $e^\Phi N\sim 1$. Also note that weak string coupling occurs for large N.

3.2 Confining solutions

Confining gauge theories are characterized in the gravity description by a unique minimum of the 4d space-time of the metric (2.2) [17]. We will elaborate on the gauge theory in section 5.

We would like to show now that the system of equations (2.9)–(2.12) admits solutions with a minimum.

Assuming $h(T)$ is a positive function, we find using eq. (2.11) that

$$\ddot\lambda>0\,.\tag{3.3}$$

Hence $\dot\lambda$ is monotonically increasing. Let us assume boundary conditions such that for small values of ρ, $\dot\lambda$ is negative. Therefore there are two possibilities: (i) $\dot\lambda$ is negative for all ρ. (ii) $\dot\lambda=0$ at some point.

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A solution in which condition ii. is satisfied implies confinement. The reason is that if $\dot{\lambda}(\rho=\rho_0)=0$, it is a minimum of $e^{2\lambda}$ and it is guaranteed that it is a *unique* minimum. We demonstrate that such solutions exist. Moreover, we argue that these solutions are generic.

We find the solutions to the equations of motions by expanding in power series around the minimum of λ, assuming λ has a minimum

$$
\begin{aligned}
\tilde{\Phi}(\rho) &= \sum_{n=0}^{\infty} \tilde{\Phi}_n (\rho-\rho_0)^n \,, \\
\tilde{\lambda}(\rho) &= \sum_{n=0}^{\infty} \tilde{\lambda}_n (\rho-\rho_0)^n \,, \\
T(\rho) &= \sum_{n=0}^{\infty} T_n (\rho-\rho_0)^n \,.
\end{aligned}
$$

In order to have a minimum of $\tilde{\lambda}$ at $\rho=\rho_0$ we set $\tilde{\lambda}_1=0$. From the zero energy constraint we get

$$
\tilde{\Phi}_1 = \pm\frac{1}{\sqrt{8}}\sqrt{e^{-4\tilde{\Phi}_0+8\tilde{\lambda}_0}\left(5-h(T_0)e^{2\tilde{\Phi}_0}+\frac{3}{16}T_0^2\right)+\frac{1}{2}T_1^2}\,, \tag{3.4}
$$

and from the equations of motion we get

$$
\begin{aligned}
\tilde{\Phi}_2 &= -\frac{e^{-4\tilde{\Phi}_0+8\tilde{\lambda}_0}}{4}\left(5-\frac{5}{2}h(T_0)e^{2\tilde{\Phi}_0}+\frac{3}{16}T_0^2\right)\,, \\
\tilde{\lambda}_2 &= \frac{1}{4}e^{-2\tilde{\Phi}_0+8\tilde{\lambda}_0}h(T_0)\,, \\
T_2 &= -e^{-4\tilde{\Phi}_0+8\tilde{\lambda}_0}\left(e^{2\tilde{\Phi}_0}h'(T_0)+\frac{3}{8}T_0\right)\,.
\end{aligned} \tag{3.5}
$$

Evidently $\tilde{\lambda}_2>0$ and hence the solution corresponds to a minimum.

There are five free parameters in the solution: $\tilde{\Phi}_0, \tilde{\lambda}_0, T_0, T_1$ and ρ_0. From (3.4) we see that some of the parameter space is excluded by the requirement of a real dilaton. We can go on and compute higher corrections in $(\rho-\rho_0)$. From the equation of motion one can see that the $\tilde{\Phi}_{n+2}, \tilde{\lambda}_{n+2}, T_{n+2}$ coefficients depend on the $\tilde{\Phi}_i, \tilde{\lambda}_i, T_i$ $(i\leq n)$ coefficients, meaning that this process can go on indefinitely.

Since the above solutions admit confinement, it is important to understand that it is not an accidental feature of the solutions. The three equations of motion have six boundary conditions. One parameter is decreased by the zero energy constraint. Another one drops out because the Hamiltonian does not depend explicitly on "time" and therefore if $\{\Phi(\rho),\lambda(\rho)\}$ is a solution then $\{\Phi(\rho+\delta\rho),\lambda(\rho+\delta\rho)\}$ is also a solution. Therefore the physical space of solutions is four dimensional. Our solution has four free parameters Φ_0, λ_0, T_0 and T_1 and accordingly has a non-zero measure in the space of solutions.

The Wilson loop analysis tells us that for large enough quark anti-quark distance L the string "spends most of it's time" near the minimum of λ which corresponds to the IR regime. The question now is how to extrapolate the solutions of above to the region along the ρ direction that corresponds in the gauge picture to the UV regime.

3.3 The flow to the UV limit

To analyze the UV limit we need to continue the solution from the minimum to the 4d boundary of the 5d space. Lets analyze the solution we found for the case of the electric magnetic branes, $h(T) = e^T + e^{-T}$. For this function we choose the solutions with $T = 0$ to make the analysis easier, though we found, numerically, similar behavior in the electric case also. Now we have only two free parameters $\tilde{\Phi}_0, \tilde{\lambda}_0$ and the solution looks like

$$\tilde{\Phi}_1 = +\frac{1}{\sqrt{8}}\sqrt{e^{-4\tilde{\Phi}_0+8\tilde{\lambda}_0}\left(5-2e^{2\tilde{\Phi}_0}\right)} \tag{3.6}$$

$$\tilde{\Phi}_2 = -\frac{1}{4}e^{-4\tilde{\Phi}_0+8\tilde{\lambda}_0}\left(5-5e^{2\tilde{\Phi}_0}\right) \tag{3.7}$$

$$\tilde{\lambda}_2 = \frac{1}{2}e^{-2\tilde{\Phi}_0+8\tilde{\lambda}_0}\,. \tag{3.8}$$

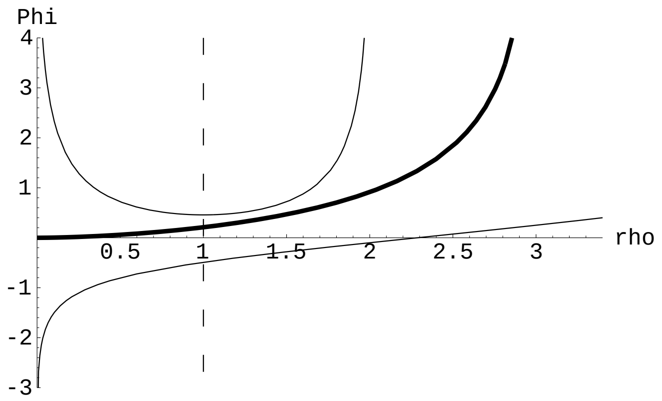

Figure 1: The dilaton behavior, for $N = 1$, at small distances. The graph describes 3 kinds of solutions of the dilaton as a function of ρ. The three possibilities are: (i) dilaton which flows to $+\infty$ in the $\rho \to 0$ limit. (ii) $\Phi(\rho = 0) = 0$ (AdS_5 solution), which corresponds to a UV fixed point and (iii) $\Phi \to -\infty$ in the UV which describes asymptotic freedom. The behavior at $\rho = 0$ is dictated by the value of Φ at the minimum of λ which we draw at $\rho = 1$.

The choice of sign in $\tilde{\Phi}_1$ is arbitrary. Changing the sign would give a mirror solution $\rho \to -\rho$. From (3.6) we get a constraint on the value of the dilaton in the minimum of λ

$$e^{2\tilde{\Phi}_0} \leq \frac{5}{2}\,. \tag{3.9}$$

Numerical analysis of the equations of motion shows us that there are three possible types of behavior for the dilaton, depending on the choice of parameters $\tilde{\Phi}_0, \tilde{\lambda}_0$. If we start with a large $\tilde{\Phi}_0$ ($e^{2\tilde{\Phi}_0}$ close to 5/2), then the dilaton would go to infinity for $\rho < \rho_0$. On the other hand, if we start with a small $\tilde{\Phi}_0$, then the dilaton would go to minus infinity. Between those two regions in the two dimensional parameter space there is a border line, at the value $e^{\tilde{\Phi}_0} \sim 1.52$, in which the dilaton $e^{\tilde{\Phi}}$ flows to 1 (see figure 1). This set of solutions with the finely tuned parameters flows to the AdS_5

solution (3.2). For the AdS_5 solution we know that the boundary is at $\rho = 0$ and $\rho = 1/u^4$. Therefore we interpret small values of ρ as large values of energy.

We can compute corrections to the exact AdS_5 solution and connect them to the solution around the minimum. If we plug ξ from (3.1) to the dilaton equation and assume that the dilaton is small ($e^{\tilde{\Phi}} \approx 1 + \tilde{\Phi}$) we get

$$4\rho^2 \ddot{\tilde{\Phi}} + 5\tilde{\Phi} = 0\,,$$

which is solved by

$$\tilde{\Phi} = C\rho^{\frac{1+\sqrt{6}}{2}}\,, \tag{3.10}$$

for any constant C. By fixing the value of C we get a $\tilde{\Phi}$ behavior which is very close to the bold line in figure 1 for small ρ. Note that since $\tilde{\Phi} \to 0$, $g^2_{YM}N \to 1$, we have a UV fixed point at finite value of the 't Hooft coupling.

In order to get asymptotic freedom we need to start with lower values of $e^{2\tilde{\Phi}_0}$ in the minimum. Numerical analysis show that those solutions flow toward $e^{2\tilde{\Phi}} \to 0$ in the UV limit. At the moment we do not know the analytical behavior of Φ and λ. and therefore we can not extract the β function.

4. Validity of the solutions

The solutions of the effective Type 0 equations of motion can be trusted only if certain conditions are obeyed. Basically one has to check that the string theory, that leads to the low energy effective action whose classical configurations we discuss, is consistent. In addition, higher order string perturbations and sub-leading α' corrections should be under control. On top of these conditions one has to check whether the correspondence to a "dual gauge theory" on the boundary in the spirit of the AdS/CFT correspondence is consistent. We discuss here the former question and the later will be addressed in section 5.

- The type 0 non critical string theory is consistent only provided that the torus partition function is modular invariant and that tadpole of the massless fields are canceled out. The critical type 0 string theory, namely the theory derived using a diagonal GSO projection, was shown to be modular invariant. We have not performed an explicit computation in the present non-critical theory. However it seems that the same structure of a modular invariant partition function holds also for our case[16]. Modular invariance was argued also in [2].

  In the type 0 theory there are potentially tachyon and dilaton tadpoles. The critical type 0 theory discussed in [6] as well as the non-critical one [2] are free from dilaton tadpoles. This is a statement about the leading behavior of the string theory. In fact one has also to assure that dilaton tadpoles are not generated even in the sub-leading string corrections[18]. At present it is not clear to us whether the type 0 models obey this stronger condition.

- Another necessary condition for the consistency of the non-critical string theory is that the tachyon T looses its tachyonic nature. In an AdS background of radius R the requirement is that $m_T^2 \geq -4/R^2$. For the CFT that is associated with the critical dyonic type 0 theory [10] the condition takes the form $4 + 16f'^2(0) \geq \sqrt{2Q}e^{\Phi/2}$ where Φ is the value of the dilaton for that model. In a general background the stability condition requires that $m_T^2 \geq c\mathcal{R}$, where c is a constant of order one. In the present case we take the requirement that $m_T^2 \geq 0$. Using (2.5) this translates into

$$e^{2\tilde{\Phi}} \geq \frac{3}{8h''(T)}\,, \tag{4.1}$$

  which is obeyed by the solutions that flow to the AdS_5. Note that one has to require the positivity of m_T^2 everywhere along the ρ coordinate and not only on the surface of the the five dimensional space. Therefore, the solutions with asymptotic freedom (small dilaton) do not obey this condition. A solution with asymptotic freedom can be stable with a non constant tachyon if $h''(T)$ diverges in the UV. There might be solutions of the pure electric branes which obey this condition.

- Non critical string theory is believed to be inconsistent for $c > 1$ and $c < c_{crit}$. Our analysis is focused on $d = c = 4$. The obvious question is whether one can make sense out of such a setup. Polyakov [2] conjectured that for non flat $d+1$ dimensions, namely d flat directions and one Liouville direction, the instability of the theory may be cured in the presence of non zero R-R background. We argued above, following [6], that the mass of the tachyon can be shifted so that $m_{Teff} > 0$ due to the coupling with the R-R field even in the non-critical dimension of $d = 4$. Since the instability of the theories at the "forbidden zone" past the $c = 1$ barrier, manifest itself in the tachyonic behavior, the shift of m_T^2 may render the theory at $d = 4$ into a consistent one. It is important to assure that even sub-leading string correction do not introduce other tachyonic modes by shifting the masses of massless or massive modes to tachyonic ones [18].

- The solutions of the equations of motion are reliable only provided that the modifications of the effective action due to higher order string loop corrections are negligible. For that, one has to insure that the string coupling $e^{\Phi} = \frac{1}{N}e^{\tilde{\Phi}}$ is weak. Indeed, the confining solutions which obey (3.9) and have large enough N, satisfy this condition for $\rho < \rho_0$. Note that although e^{Φ} diverges at $\rho \to \infty$ and therefore the geometry at large values of ρ will be modified by loop corrections, this region is not important since the string associated with the Wilson loop does not probe this region (see section 5.4).

- The gravitational sector of the low energy effective field theory (2.1) is a valid approximation to the full gravitational effective action only provided that higher order curvature terms are negligible. As was argued in [6] the world sheet supersymmetry, even in the absence of space-time supersymmetry restricts the corrections to the type 0 effective action to be identical to those of the type II. In the gravitational sector the first corrections are proportional to $\alpha'^3 R^4$, and only the Weyl tensor contributes to the R^4 term. It turns out that the five dimensional metric of the general form (2.2), not necessarily of an AdS_5 form, has a *vanishing Weyl tensor* so that the first order correction vanishes. Since the full list of higher order curvature corrections is not known to us, to be on the safe side, we would like to impose a restriction that the Ricci scalar is small in the string frame.

  The curvature (in the string frame) associated with the solution (3.4), (3.5) is given, near the minimum, by

  $$R = -8e^{2\tilde{\Phi}_0} + O(\rho - \rho_0)\,. \tag{4.2}$$

  Hence, for small Φ_0 we are guaranteed not to have significant modifications to the gravitational effective action. Unfortunately, a small Φ_0 is restricted by the requirement for a stable tachyon (4.1). We believe that this contradiction appears because both conditions are too strict. The condition for the small Ricci scalar is too strict because it seems that the first higher order curvature corrections depend only on the vanishing Weyl tensor. The condition for the stable tachyon we have used does not take into account the fact that the metric has a negative curvature. On top of that solutions with divergent $h''(T)$ may help stabilize the tachyon.

- The stability of the stack of N $D3$ branes. Since there is no space-time supersymmetry the D branes are not BPS states and there is a-priori no reason that there is no force between them. Recently, stable non-BPS states where studied in various setups [19]. Even though we believe that the two phenomena may be related the direct implications of [19] to the present case are still unclear to us. Note that the bosonic degrees of freedom of the type 0 are identical to those of the type II apart from the fact that in the former case there is a doublet of R-R fields. The computation of the interaction energy between two parallel $D3$ branes was presented in [6] for the case of a single type of $D3$ branes and for electric-magnetic branes. It seems that the same features show up also in the non-critical type 0 string. It is therefore our believe that there might be stable stacks of N electric (or magnetic) $D3$ brane as well as electric-magnetic stacks of branes. Recall that the stability at the level of subleading string corrections relates to the issue of possible generation of dilaton tadpole [18].

In fact the stability in the non-critical case may be in a better shape than the stability in the critical case, because in the later case there are flat directions [11, 13]. The 5d non-critical theory has only the Liouville direction perpendicular to the branes, so there are no flat direction in the first place. In the language of the gauge fields this translates into the fact that there are no (adjoint) scalar fields and thus the SU(N) gauge symmetry cannot be spontaneously broken.

- Unlike the confining solutions of [20, 25] which have a "cigar-like" shape, the solutions constructed in section 3, that admit a minimum value of $e^{2\lambda}$, are characterized generically by having two boundaries.[1] The boundary at $\rho = 0$ where the 4d field theory exists as well as an additional one at $\rho \to \infty$. In terms of the consistency of the supergravity solution such a scenario is admissible, however it may be problematic to the dual gauge theory description. The point is that to guarantee a unitary gauge theory on the boundary one has to assure that signals cannot propagate in finite time to the other boundary. This condition translates into $\int_0^\infty d\tau e^{-2\lambda(\tau)} \to \infty$. To check this condition the metric solution has to be reliable not only in the region between $\rho = 0$ and ρ_0 but also all the way to $\rho \to \infty$. Since toward the later limit the dilaton blows up we are not able to assure that our solution obeys the condition.

5. The gauge theory interpretation

Now that solutions of the equations of motion were written down and their validity was considered, we face the challenge of deducing the interpretation of the solutions in terms of the four dimensional boundary gauge theory. Obviously, in the present non supersymmetric and non conformal scenario it is more difficult to argue in favor of the duality between the the supergravity solution and the gauge theory on the boundary. Nevertheless, we try to analyze the gauge interpretation following the recipe of the $AdS_5 \times S^5$ and the $\mathcal{N} = 4$ SYM correspondence.

5.1 Symmetries

- The bulk space-time effective theory of the type 0 string is not invariant under any supersymmetry transformation, therefore the "dual" gauge theory should also be a non-supersymmetric one.

- The electric solution and the electric-magnetic solution incorporate N "electric" and N electric and N magnetic non BPS $D3$ branes respectively. It is well known that the open strings between the coinciding $D3$ branes (of the same

[1]We thank J. Maldacena for pointing out this condition

type) induce SU(N) gauge fields. Hence the electric theory and the electric-magnetic theories should be invariant under local SU(N) and SU(N) $\times$ SU(N) symmetries respectively.

- Global symmetries of the boundary theory originate from isometries of the 5d space-time metric (which are in addition to the 3+1 dimensional Poincare symmetry). The confining solutions do not admit any symmetries. The AdS_5 has an SO(2, 4) isometry group which maps into the symmetries of the four dimensional conformal gauge theory.

5.2 Gauge coupling

In the absence of a tachyon field in the vacuum, the effective gauge theory on the $D3$ brane has the form of $e^{-\Phi}\mathrm{Tr}[F^2]$ so that the gauge coupling is expected to be $g^2_{YM} \sim e^{\Phi}$. It was put forward that this relation translates the evolution of the coupling constant as a function of the energy scale into the dependence of Φ on the fifth dimension. In case that the tachyon is non vanishing in the ground state, the gauge coupling is dressed $e^{\Phi} \to F(T)e^{\Phi}$. Since one can introduce a transformation of the fifth coordinate, an non ambiguous result can be derived from a "physical measurement" like the Wilson line discussed below. Unfortunately, the two prescriptions are in conflict in the critical type 0 theory [9]. Moreover, it happens in the present case even when the tachyon is zero.

In the extreme UV regime, where the expected couling is *weak* and hence the potential is expected to be $g^2_{YM}(L)N/L$, one can read the relation between the gauge coupling and the dilaton. In the case of critical strings the relation was $g^2_{YM}N = e^{\frac{1}{2}\Phi}$. In the present case it is $g^2_{YM}N = e^{\frac{4}{3}\Phi}$. We will comment about it when we will discuss the Wilson loop.

5.3 Field content

- The full set of fields on the world volume of D branes is determined usually by the vector fields generated by the open strings, the scalar fields associated with transverse free motions of the brane and the amount of supersymmetries. In the present non critical type 0 theory supersymmetry does not enforce additional fields in the adjoint representation. Moreover the Liouville direction is not a free direction and thus a massless scalar field cannot be associated with the motion along this direction. We therefore conclude that the electric theory does not include any fields apart from the gauge fields, namely, it is a pure Yang Mills theory. In terms of its symmetries, the confining solution and its flow to the UV is compatible with such a gauge scenario. On the other hand the exact AdS_5 solution dictates a conformal gauge theory. This clearly cannot be associate with the quantum Yang Mills theory. Classically, the theory is conformal invariant but clearly this is not a property of the full quantum theory.

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At present we do not know how to resolve this puzzle. In [2] it is argued that the AdS_5 solution may be associated with a UV fixed point. But since it seems to be an exact solution for the whole range of ρ this conjecture is not justified.

- The situation in the electric-magnetic theory is different. As was shown in [21] the open strings between the electric and magnetic D branes constitute a matter field on the world volume in the $(N, \bar{N}) \oplus (\bar{N}, N)$ representation. Unlike the conformal models of [10] where the global symmetry enforced a multiplicity of four such representations, in the present case since there is no SO(6) global symmetry, there is only one such matter field representation. The action which includes those fields should not be invariant under any additional global symmetry.

5.4 The Wilson loop

A very important tool to translate the super gravity solution into the gauge field language is the computation of the Wilson loop introduced in [22, 23]. By now the Wilson loop and similar objects were computed in various setups, [24, 25]. In particular in [17] a necessary condition for a confining behavior was derived. We define the functions

$$f^2(u) \equiv G_{tt}(u)G_{xx}(u)$$
$$g^2(u) \equiv G_{tt}(u)G_{uu}(u)$$

(where u is the fifth coordinate) in terms of which the Nambu Goto action for the fundamental string takes the form

$$S_{F1} = \int d^2\sigma\sqrt{h} = T\int d\sigma\sqrt{f^2(u) + g^2(u)\,(\partial_x u)^2}\,. \tag{5.1}$$

Then the condition for confinement is that f admits a minimum and the corresponding string tension is $\sigma = f(u_{min})$. The setup behind this Nambu-Goto action is such that the quark anti-quark pair are situated at $u \to \infty$ and the string connecting them stretches in the domain of $\infty > u \geq u_{min}$. In the coordinate assignment of section 2 (2.2) the boundary where the external charges are put is at $\rho = 0$. Note also that in the solutions discussed in section 2, $f = e^{2\lambda}$.

In the AdS_5 solution one finds that the quark anti-quark potential takes the form $E = c/L$ where c is a numerical factor.

For the AdS_5 fixed point of (3.10) we are unable to calculate the exact Wilson loop for this metric but we can follow the approximation done in [7] and assuming $e^{\frac{4}{3}\tilde{\Phi}}$ is ρ independent for small ρ. Then we get

$$E \sim \frac{e^{\frac{4}{3}\tilde{\Phi}}}{L}\,.$$

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Matching this result to that in the neighborhood of a UV fixed point of a gauge theory namely $E \sim g_{YM}^2 N/L$ one can read the relation between the running gauge coupling and the dilaton.

This is a manifestation of Polyakov's suggestion that the AdS_5 solution describes only the extreme UV behavior of the gauge theory [2].

Two remarks are in order, (i) the relation between the dilaton and g_{YM}^2 does not agree with the basic relation $e^{\Phi} \sim g_{YM}^2$ that follows from the open string origin of the gauge fields. (ii) It is amusing that for a non-critical theory in 6 dimensions one does find agreement between the naive assignment and the one that follows from the Wilson loop.

In the case of asymptotically free solutions, it is very difficult to extract the the exact behavior of the gauge coupling. The reason is that the metric in this region is found by numerical integration. In principle, this is enough to calculate the Wilson loop. Practically, it is almost hopeless. For the determination of the behavior in this regime, we will need analytical solutions, which are not at hand at the moment.

Next we address the Wilson loop in the IR regime. Though our solution (3.4), (3.5) is valid at a limited region in the interior of the interval $\rho = [0, \infty)$, the IR behavior is determined by this regime of the solution.

In particular, the space-time 4d part of the metric (2.2) admits a minimum at some point $\rho = \rho_0$. As a result [17] a confining potential exists

$$V = \sigma r \,,$$

with the following string tension

$$\sigma \sim \min \frac{1}{N} e^{2\tilde{\lambda}} = \frac{1}{N} e^{2\tilde{\lambda}_0} \,. \tag{5.2}$$

It is important to note that since the minimum of λ is unique, it is guaranteed that higher order corrections wouldn't spoil the confining nature of the solution.

Note that though it seems that the string tension behaves as (5.2) $1/N$ it is not the case. We may choose the value of $\tilde{\lambda}_0$ such that $\sigma \sim \Lambda_{QCD}^2$ (in $\alpha' = 1$ units).

5.5 The 't Hooft loop

Confinement of quarks should be followed by screening of magnetic monopoles. We would like to check whether our confining solutions obey this property. We use the recipe of refs. [24, 25].

The monopole anti-monopole potential is given by the $D1$ action

$$S_{D1} = \int d^2\sigma e^{-\Phi} \sqrt{h} = T \int d\sigma e^{-\Phi} \sqrt{f^2(u) + g^2(u)\,(\partial_x u)^2} \,.$$

Thus the question of screening versus confinement in this case depends on the behavior of $e^{-\Phi} f = e^{-\Phi + 2\lambda}$. Though we can't prove that magnetic confinement is excluded, the numerical behavior of Φ and λ tells that for quark confining solutions which flow to UV fixed point or to UV free behavior, we *do have* magnetic screening.

5.6 Zig Zag invariance

Polyakov [2] stated the invariance of the Wilson loop under the Zig Zag transformation as a necessary condition for any string theory description of a gauge theory. The condition translates to the following requirement on the metric

$$e^{\lambda(\tau_*)} = 0 \qquad \text{or} \quad \infty$$

where τ_* is the value of τ at the boundary field theory. This condition follows from the fact that the solutions found for Φ and λ do not extremize the boundary term in the effective action[2] and thus the $D3$ branes are driven to either $\tau = 0$ or $\tau = \infty$. Polyakov further advocated the option of $e^{\lambda(\tau_*)} = \infty$ for which massive boundary states vanish, and the momentum of massless ones is not restricted. The only massless states should be the gauge fields. As argued in 5.3 indeed there are no additional massless scalar fields on the boundary and in the electric case no additional fields at all.

The AdS_5 solutions, both the exact (section 3.1) and the UV fixed point (section 3.3), obey this condition since for those cases $\lambda \to \infty$. The flow of the confining solution to a solution with an "asymptotic freedom" does not obey this condition since $e^{\lambda\tau_*}$ is a constant.

6. Summary and discussion

Whereas the duality between 4d SYM in the large N limit and the string theory on an $AdS_5 \times S^5$ background is by now well established, there are only first hints that a similar correspondence may be applicable also to the pure YM theory. Moreover, it seems now that the more promising avenue to construct a string theory of strong interaction is via a gauge/gravity duality. Our work is aimed at improving the understanding of this approach.

Motivated by the required form of the space time metric that yields confining Wilson loop [17] we searched for solutions of the equations of motion for which λ has a minimum. A class of such solutions was identified. We showed also that the solution around the region of the minimum is smoothly connected to the region of small values of ρ and found that in the region of small $e^{2\tilde{\Phi}_0}$, $\tilde{\Phi} \to -\infty$. Therefore, the corresponding gauge theories are UV free.

It is interesting to check whether the flow towards the UV is logarithmic as a function of the energy scale. While in other similar cases [7, 8, 12] it was shown that this is indeed the case, we haven't find such analytical solutions in the present case. Nevertheless, the numerical study does not exclude such a possibility. The analysis was performed in the "electric-magnetic theory" only, where $T = 0$. It is important to analyze the UV regime in the electric theory which describes pure Yang-Mills. We postpone these issues for the future. Another interesting direction is to extract

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the glueballs mass spectrum and to compare it to lattice results. For this one has to know the full behavior of the metric.

A point that was not emphasized enough in the study of the type 0 string is that the large N limit seems to be different then the one implemented in the original duality of Maldacena [3]. To guarantee the consistency of the solutions, namely small string coupling and small curvature large N was useful but not large $g_s N$. In fact we found that what was needed is $g_s N > 3/16$ to guarantee the removal of the tachyonic behavior and $g_s N < 1$ to assure small scalar curvature in the string frame.

On the route to the solutions several assumptions were made which deserves further justification. In particular an explicit computation of the torus partition function has to be performed so that modular invariance could be checked. Additional checks about possible generation of tadpoles of massless particles due to sub leading correction are also needed.

An interesting issue which has not been investigated thoroughly enough is the relation between the type 0 D branes that were used in the present work (and also in the studies made in the critical setting) and the the stable non BPS D branes discussed in [19]. In particular It seems to us that the non trivial tachyon profiles discovered recently [26] may play an important role also in the string solutions that corresponds to gauge dynamics.

Recently, several authors discussed super-gravity solutions of type IIB, in which the matter fields are massive [27, 28, 29]. As expected from the field theory side, these solutions also admits confinement. It would be interesting to compare their results to ours.

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A. Asymptotic behavior

While it is difficult to find the precise behavior of the solutions at large ρ we can negate some possibilities. Since $\dot{\lambda}$ is monotonically increasing, it might approach some limit at $\rho \to \infty$. We would like to show that while it is impossible in the theory which lives on the electric-magnetic sets of branes, there is a room for such scenario in the electric branes theory (pure Yang-Mills).

Let us first assume zero tachyon solution with the following large ρ behavior

$$\lambda \to \lambda_\infty \rho \,,$$
$$\Phi \to \Phi_\infty \rho \,.$$

For the fall-off of the exponentials in (2.10), (2.11) we impose

$$-2\Phi_\infty + 8\lambda_\infty < 0 \,,$$
$$-4\Phi_\infty + 8\lambda_\infty < 0 \,.$$

In addition

$$\lambda_\infty > 0 \,,$$

and from the Hamiltonian

$$-4{\Phi_\infty}^2 + 16\Phi_\infty\lambda_\infty - 12{\lambda_\infty}^2 = 0 \,,$$

which cannot be satisfied with the above constraints.

In the case of non-zero tachyon and $h(T) = e^{-T}$ we may assume a similar asymptotic behavior for the tachyon

$$T \to T_\infty \rho \,.$$

Now, the requirements from the exponentials are

$$-T_\infty - 2\Phi_\infty + 8\lambda_\infty < 0 \,,$$
$$-4\Phi_\infty + 8\lambda_\infty < 0 \,.$$

and from the Hamiltonian

$$-4{\Phi_\infty}^2 + 16\Phi_\infty\lambda_\infty - 12{\lambda_\infty}^2 + \frac{1}{4}{T_\infty}^2 = 0 \,. \tag{A.1}$$

These conditions are satisfied in the range $\frac{1}{4}\Phi_\infty > \lambda_\infty > 0$.

Therefore when a tachyon is included, the asymptotic behavior of Φ, λ and T can be linear at large values of ρ, while when $T = 0$ (and $h(T)$ is an even function) the derivative of Φ or λ must diverge.

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